

Liberal Non-Interference Under a Status Quo*

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Abstract

Social welfare relations abstract away from the historical context when evaluating welfare profiles. We study the implications of liberal non-interference for *contextualised social welfare relations* that compare welfare profiles relative to a status quo. We find that a liberal principle that protects individuals against damage, together with standard efficiency, anonymity, continuity, and informational properties, characterises a *maximin difference relation* that is egalitarian with respect to welfare differences rather than welfare levels. Absolute priority is given to individuals who are worst-off relative to the status quo. Each individual is guaranteed a veto over departures from the status quo which reduce their welfare, irrespective of the gains such departures would generate for others.

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1 Introduction

Social welfare relations (henceforth, SWRs) evaluate welfare profiles in isolation from the historical context. One welfare profile is compared to another based only on the information contained in those profiles, independent of any information about the history that led to them. This makes standard SWRs unable to capture objectives of a policy maker that might be history-dependent. Consider, for example, one who seeks to evaluate welfare gains differently from welfare losses, or ensure that their evaluations are sensitive to the historically-acceptable levels of inequality, or needs to make reparations for historical injustices. Standard SWRs would not be able to capture these policy objectives.

This paper studies the implications of a liberal principle of non-interference for *contextualised SWRs* that incorporate a historical context for social evaluation in the form of a status quo welfare profile. Specifically, a contextualised SWR is a binary relation over *contextualised welfare profiles* (a, s) where a is a welfare profile and s a status quo. Adding a status quo makes contextualised SWRs able to capture history-dependent policy objectives. Welfare gains and losses could have a meaningful interpretation when calculated with respect to status quo welfare. Welfare profiles that are within historically-acceptable levels of inequality could be defined as those where welfare differences between individuals do not exceed those in the status quo profile. Individuals entitled to reparations could be identified as those who have a status quo welfare below a certain threshold.

Liberal non-interference principles give conditions under which individuals have a right not to be interfered with by others. The principle we use for our characterisation – the *Individual Damage Principle* – only requires that an individual has a right not to be *punished* when they have *already suffered a strict welfare loss in a way in which no one else was affected*. The principle does not protect the rights of any individual to be rewarded, nor the right to not be interfered with when they have enjoyed a welfare gain, or when someone else was affected.

We find that this principle, together with standard efficiency, anonymity, continuity, and informational properties, characterises a *maximin difference relation* that ranks one contextualised welfare profile (a, s) weakly above another (a', s) under the same status quo s if and only if the minimum welfare difference for an individual between a and s is weakly higher than that under a' and s . The maximin difference relation equalises welfare differences relative to the status quo, rather than welfare levels as for the standard maximin SWR. Equalising welfare differences recovers equalising welfare levels as a special case only when all individuals have the same welfare in the status quo.

The maximin difference relation gives absolute priority to individuals i who are worst-off in terms of their welfare difference with the status quo $a_i - s_i$. This implies a strong protection of individual baseline welfare. Each individual is guaranteed a veto against any

departure from the status quo that would lead them to a welfare loss, regardless of the potential gains to the others. Consider, for example, two individuals under a status quo $s = (10, 0)$. According to the maximin difference relation, $((10, 0), s)$ is strictly better than $((10 - \varepsilon, \frac{1}{\varepsilon}), s)$ for all $\varepsilon > 0$. Individual 1 is able to veto any redistribution of their welfare to individual 2 even as $\varepsilon \rightarrow 0$ where the welfare loss to 1 becomes arbitrarily small and the welfare gain to 2 arbitrarily large.

Moreover, in applications where welfare levels are interpersonally comparable, equalising welfare differences can preserve pre-existing inequalities in welfare levels, even when only welfare gains are considered. The maximin difference relation, for example, requires that $((10 + \frac{1}{\varepsilon}, \varepsilon), s)$ is strictly better than $((10 + \frac{\varepsilon}{2}, 10 + \frac{\varepsilon}{2}), s)$ for all $\varepsilon \in (0, 1)$ where $s = (10, 0)$. Yet, as $\varepsilon \rightarrow 0$, the first profile $((10 + \frac{1}{\varepsilon}, \varepsilon), s)$ approaches perfect inequality, whereas the second $((10 + \frac{\varepsilon}{2}, 10 + \frac{\varepsilon}{2}), s)$ is perfectly equal.

1.1 Related literature

We incorporate into the standard SWR framework the historical context in the form of a status quo welfare profile. This is in contrast to most welfarist social choice where the historical context is abstracted away, as is the case in the social welfare function (e.g. Bergson 1938; Samuelson 1947; Ng 2004), social welfare functional (e.g. Arrow 1963; Sen 1970; d’Aspremont and Gevers 2002), and collective choice (e.g. Peters and Wakker 1991; Ok and Zhou 1999; Mariotti 2000) frameworks. Moreover, our approach differs from cooperative bargaining problems (e.g. Nash 1950), cooperative rationing problems (e.g. Chun and Thomson 1992; Herrero 1997; Mariotti and Villar 2005), and Hard Problems (e.g. Cheung et al. 2026), in that we do not just study the choice of one welfare profile given a status quo, but are able to rank any such profile with another.

Bossert and Sprumont (2003) study generalised social choice functions that take a feasible set of alternatives, together with a reference alternative, to give a subset of that feasible set. Conditions on generalised social choice functions are given such that there exists a profile of preferences where each of the chosen alternatives are Pareto efficient and at least as good as the status quo. Similarly, Maniquet and Moramarco (2026) characterise a number of well-being measures defined over a domain of consumption bundles, preferences over bundles, and reference bundles for each individual. We instead study the implications of liberal non-interference on social preferences over pairs of alternatives and status quos.

The contextualised SWRs we propose are formally equivalent to difference preorders that have been used to characterise utilitarian and quasi-utilitarian SWRs (e.g. Dyer and Sarin 1978, 1979a,b; Harvey 1999; Harvey and Østerdal 2010; Pivato 2015; see also Harsanyi (1955)). These characterisations use concatenation conditions which require that if a welfare improvement from a profile a to b and from b to c are respectively just as good as improvements from a' to b' and b' to c' , then the overall improvement from a to c should

be just as good as one from a' to c' . We do not require any concatenation condition, but instead use a liberal non-interference principle to characterise a maximin difference relation.

Contextualised SWRs are also formally equivalent to the quaternary relations that have been used to characterise utility difference representations of the form, $(a, b) \succsim (a', b')$ if and only if $u(a) - u(b) \geq u(a') - u(b')$ for some utility function u (e.g. Krantz et al. 1971; Shapley 1975; Wakker 1988; see Köbberling 2006 for a survey). These characterisations also depend on concatenation conditions, except for Gerasimou (2021) that proposes an ordinal approach to preference intensities that does not require concatenation (see also Baccelli 2024). Our results instead use conditions on these relations that have normative interpretations.

Finally, the liberal principle of non-interference we use is due to Mariotti and Veneziani (2009). See also the subsequent literature studying liberal principles of distributive justice (e.g. Lombardi et al. 2016; Mariotti and Veneziani 2013; 2017; 2018; Alcantud et al. 2022). Our paper generalises this liberal non-interference principle to contextualised SWRs. We compare in more detail our main characterisation with that of Mariotti and Veneziani (2009) and Lombardi et al. (2016) in Section 3 below.

This paper is structured as follows. Section 2 presents the framework. Section 3 derives the main characterisation. Section 4 concludes.

2 Framework

Let $\mathcal{N}$ be any finite set of individuals. Denote $N > 1$ the cardinality of $\mathcal{N}$. Let $\mathbb{R}$ be the set of real numbers. A *contextualised welfare profile*, or simply a *profile*, is any $(a, s) \in \mathbb{R}^{2N}$ where $a \in \mathbb{R}^N$ is a *welfare profile* and $s \in \mathbb{R}^N$ a *status quo* welfare profile for the N individuals. We may interpret ‘welfare’ as utility, a share of a divisible good, an index of access to primary goods, among many other things, as long as it can be represented as a real number.

Definition 2.1. A *contextualised social welfare relation* $\succsim$ on $\mathbb{R}^{2N}$ is a binary relation such that for any profiles $(a, s), (b, t) \in \mathbb{R}^{2N}$, $(a, s) \succsim (b, t)$ means the welfare profile a given status quo s is at least as socially preferred to b given status quo t .

Denote $\succ$ and $\sim$ the asymmetric and symmetric components respectively. Contextualised SWRs incorporate the historical context into social welfare evaluation in the form of a status quo welfare profile.¹ A contextualised SWR is *s-Reflexive* when for all $(a, s) \in \mathbb{R}^{2N}$, $(a, s) \succsim (a, s)$; *s-Transitive* when for all $(a, s), (b, s), (c, s) \in \mathbb{R}^{2N}$, if $(a, s) \succsim (b, s)$ and $(b, s) \succsim (c, s)$, then $(a, s) \succsim (c, s)$; and *s-Complete* when for all $(a, s), (b, s) \in \mathbb{R}^{2N}$, if

¹Standard SWRs are recovered when for all $(a, s), (b, t), (a, s'), (b, t') \in \mathbb{R}^{2N}$, $(a, s) \succsim (b, t)$ if and only if $(a, s') \succsim (b, t')$.

$(a, s) \neq (b, s)$, then $(a, s) \succsim (b, s)$ or $(a, s) \precsim (b, s)$.²

We focus on contextualised SWRs that satisfy s -Reflexivity, s -Transitivity, and s -Completeness. We call these *contextualised social welfare rankings*. This means that we are concerned only with social welfare comparisons under the same status quo. Such comparisons are appropriate for actual policy evaluations, where the status quo is indeed the same. Moreover, comparisons under different status quos would not be appropriate for our liberal non-interference principle which focuses only on perturbations on the welfare of a single individual under a fixed social situation. Changing the status quo would change the social situation. This would mean that the individual protections against damage as required by our principle would no longer be well-defined because we would be comparing welfare perturbations under potentially very different social situations.

Contextualised social welfare rankings can account for a number of history-dependent policy objectives.

A policy maker may seek to evaluate welfare gains differently from welfare losses relative to the status quo. For any profile $(x, s) \in \mathbb{R}^{2N}$, let the *welfare-difference profile* $x - s \in \mathbb{R}^N$ be the profile such that $(x - s)_i = x_i - s_i$ for all $i \in \mathcal{N}$.

Example 2.1. (*Welfare-Difference Evaluation*).

$$(a, s) \succsim (b, s) \text{ if and only if } W(a - s) \geq W(b - s)$$

for some social welfare function W over welfare-difference profiles.

Sensitivity to historically-acceptable levels of welfare inequality can also be taken into account by contextualised social welfare rankings. For any profile $(x, s) \in \mathbb{R}^{2N}$, let the *inequality-difference profile* $d(x, s) \in \mathbb{R}^{\frac{N(N-1)}{2}}$ be such that $d_{(i,j)}(x, s) = |x_i - x_j| - |s_i - s_j|$ for all $i, j \in \mathcal{N}$. Each component $d_{(i,j)}(x, s)$ captures the difference in the inequality between individuals i and j under the welfare profile x and that under the status quo s .

Example 2.2. (*Inequality-Sensitive Evaluation*).

$$(a, s) \succsim (b, s) \text{ if and only if } W(a, d(a, s)) \geq W(b, d(b, s))$$

for some social welfare function W over welfare profiles together with their corresponding inequality-difference profiles.

Reparations for historical injustices in the form of having a status quo welfare lower than a given threshold can also be incorporated by contextualised social welfare rankings. Let $\rho \in \mathbb{R}$ be a level of status quo welfare under which an individual is eligible for reparations.

²We use the ‘ s ’ in front of property names to emphasise that they only hold for profiles with the same status quo.

For any profile $(a, s) \in \mathbb{R}^{2N}$, let a_s^ρ be the profile a with only components a_i such that $s_i \leq \rho$, and let $a_s^{-\rho}$ be the profile with only the components a_i such that $s_i > \rho$.

Example 2.3. (*Reparations*).

$$(a, s) \succsim (b, s) \text{ if and only if } W(r(a_s^\rho), a_s^{-\rho}) \geq W(r(b_s^\rho), b_s^{-\rho})$$

for some social welfare function r only for individuals entitled to reparations, and W which aggregates the social welfare of those who are entitled to reparations with the welfare of those who are not.

A *positive affine transformation* is a mapping $\tau : \mathbb{R}^N \rightarrow \mathbb{R}^N$ such that, for some real numbers $\alpha_i > 0$ and β_i , $\tau_i(x) = \alpha_i x + \beta_i$ for all $i \in \mathcal{N}$. A positive affine transformation is *unit comparable* if $\alpha_i = \alpha_j$ for all $i, j \in \mathcal{N}$ and *ratio-scale measurable* if $\beta_i = \beta_j = 0$ for all $i, j \in \mathcal{N}$. A *permutation* is a bijection, $\pi : \mathcal{N} \rightarrow \mathcal{N}$, such that for all $a \in \mathbb{R}^N$, $a^\pi := (a_{\pi(i)})_{i \in \mathcal{N}}$. For any $a \in \mathbb{R}^N$, denote $\bar{a} \in \mathbb{R}^N$ to be a permutation of a such that the components are in increasing order. Moreover, denote $\mathbf{0}$ the zero vector, $\|\cdot\|$ the Euclidean norm, $B(x, \varepsilon) := \{y \in \mathbb{R}^N \mid \|x - y\| < \varepsilon\}$ an open ball of radius $\varepsilon > 0$ centred at $x \in \mathbb{R}^N$, and $\gg$ the vector inequality on $\mathbb{R}^N$ such that $x \gg y$ if and only if $x_i > y_i$ for all $i \in \mathcal{N}$.

3 Characterisation

We study contextualised social welfare rankings that satisfy a liberal principle of non-interference. This principle requires that if someone suffers a welfare loss in a way in which no one else is affected, then she should not be punished further. Such a punishment would interfere with this individual in a way that is unjustified. This is a weak form of liberal non-interference which constrains social evaluations in a rather limited set of scenarios. We also only require the principle to hold for profiles with the same status quo. Formally:

s-Individual Damage Principle (s-IDP): For all $(a, s), (b, s), (a', s), (b', s) \in \mathbb{R}^{2N}$, if $(a, s) \succ (b, s)$, and

$$\begin{aligned} a'_i &< a_i \text{ and } b'_i < b_i \text{ for some individual } i \in \mathcal{N} \text{ and,} \\ a'_j &= a_j \text{ and } b'_j = b_j \text{ for every other individual } j \neq i, \end{aligned}$$

then $(b', s) \not\succ (a', s)$ whenever $a'_i > b'_i$.

The Individual Damage Principle for standard SWRs is due to Mariotti and Veneziani (2009).³ The *s*-Individual Damage Principle holds when there is one individual i who suffers

³See also Mariotti and Veneziani (2013; 2017) for detailed discussions.

strict welfare losses from (a, s) and (b, s) to (a', s) and (b', s) , respectively, but no one else is affected. An example is an individual who makes a bad choice that harms themselves but is harmless to others. The principle requires that if for whatever reason $(a, s) \succ (b, s)$, and i has higher welfare under (a', s) than (b', s) , then strict social preferences should not switch against i , $(b', s) \succ (a', s)$. Given that nobody else was affected, the individual has a right not to be punished further. Comparing profiles with the same status quo ensures that the principle only holds under a fixed social situation where there is a single individual who suffers damage that affects no one else. Comparing profiles with different status quos would mean comparing different social situations under which there may be reasons compatible with liberalism to interfere.

We impose standard requirements of efficiency and fairness under the same status quo.

s-Weak Pareto Efficiency: For all $(a, s), (b, s) \in \mathbb{R}^{2N}$, if $a \gg b$, then $(a, s) \succ (b, s)$.

s-Anonymity: For all $(a, s), (b, s) \in \mathbb{R}^{2N}$, if there exist $i, j \in \mathcal{N}$ such that $a_i = b_j$, $a_j = b_i$, $s_i = s_j$, and, for all other $k \notin \{i, j\}$, and $a_k = b_k$, then $(a, s) \sim (b, s)$.

s-Weak Pareto Efficiency requires that one profile (a, s) is strictly socially preferred to another (a', s) with the same status quo when the welfare of every individual under a is strictly higher than that under a' . *s-Anonymity* requires that the identity of the individuals does not matter: together with *s-Transitivity*, it implies that for any $(a, s), (b, s) \in \mathbb{R}^{2N}$ such that $b = a^\pi$ for any permutation π and $s_i = s_j$ for all $i, j \in \mathcal{N}$, we have that $(a, s) \sim (b, s)$.

We also require contextualised social welfare rankings to satisfy continuity under the same status quo.

s-Continuity: For all $(a, s) \in \mathbb{R}^{2N}$, the upper and lower contour sets $\{(b, s) \in \mathbb{R}^{2N} \mid (b, s) \succeq (a, s)\}$ and $\{(b, s) \in \mathbb{R}^{2N} \mid (b, s) \preceq (a, s)\}$ are closed.

Finally, we require contextualised social welfare rankings to allow for welfare to be at least cardinally measurable over profiles that have the same status quo. Cardinal measurability is in a sense a minimal informational requirement in order for contextualised social welfare rankings to adequately account for the historical context. It allows for comparing welfare differences from the status quo to be meaningful, without which the welfare difference between a welfare profile a and status quo s could not be meaningfully compared with the welfare difference under another welfare profile a' . Cardinal measurability makes comparing relative welfare differences of a single individual meaningful, but not of comparing neither relative welfare differences nor relative welfare levels across individuals meaningful.

s-Cardinal Measurability: For all $(a, s), (b, s) \in \mathbb{R}^{2N}$, $(a, s) \succeq (b, s)$ if and only if $(\tau(a), \tau(s)) \succeq (\tau(b), \tau(s))$ for all τ positive affine transformations.

However, as Proposition 1 shows, s -Cardinal Measurability is by itself not enough to use contextualised social welfare relations for social welfare evaluation even under s -Weak Pareto Efficiency, s -Anonymity, and s -Continuity.

Proposition 1. *There is no contextualised social welfare ranking that satisfies s -Weak Pareto Efficiency, s -Anonymity, s -Continuity, and s -Cardinal Measurability.*

Proof. Suppose by contradiction that there is a contextualised social welfare ranking $\succsim$ that satisfies s -Weak Pareto Efficiency, s -Anonymity, s -Continuity, and s -Cardinal Measurability. Let R be a binary relation over $\mathbb{R}^N$ such that for all $a, b \in \mathbb{R}^N$, aRb if and only if $(a, \mathbf{0}) \succsim (b, \mathbf{0})$. Denote P and I the asymmetric and symmetric components respectively.

Because $\succsim$ satisfies s -Reflexivity, s -Transitivity, and s -Completeness, R is a weak order. Similarly, for the remaining properties satisfied by $\succsim$, we have that R satisfies *Weak Pareto Efficiency* in that for all $a, b \in \mathbb{R}^N$, if $a \gg b$, then aPb ; *Anonymity* in that for all $a, b \in \mathbb{R}^N$, if there exist $i, j \in \mathcal{N}$ such that $a_i = b_j$, $a_j = b_i$, and, for all other $k \notin \{i, j\}$, $a_k = b_k$, then aIb ; and *Continuity* in that for all $a \in \mathbb{R}^N$, the upper and lower contour sets $\{b \in \mathbb{R}^N \mid bRa\}$ and $\{b \in \mathbb{R}^N \mid aRb\}$ are closed. Moreover, for every ratio-scale transformation τ , $\tau(\mathbf{0}) = \mathbf{0}$. Hence, s -Cardinal Measurability implies aRb if and only if $\tau(a)R\tau(b)$. That is, the s -Cardinal Measurability of $\succsim$ implies the *Ratio-Scale Measurability* of R .

Therefore, by Theorem 6 of Tsui and Weymark (1997) (see also Nebel 2023), there is $i \in \mathcal{N}$ such that for all $a, b \in \mathbb{R}^N$, aRb if and only if $a_i \geq b_i$. However, this means that R violates Anonymity, a contradiction. $\square$

This impossibility is overcome when we allow for welfare to not just be cardinally measurable, but also unit comparable over profiles that have the same status quo.

s -Unit-Comparable Cardinal Measurability: For all $(a, s), (b, s) \in \mathbb{R}^{2N}$, $(a, s) \succsim (b, s)$ if and only if $(\tau(a), \tau(s)) \succsim (\tau(b), \tau(s))$ for all τ unit-comparable positive affine transformations.

We find that any contextualised social welfare ranking that satisfies the s -Individual Damage Principle, s -Weak Pareto Efficiency, s -Anonymity, s -Continuity, and s -Unit-Comparable Cardinal Measurability must be the maximin difference relation.

Definition 3.1. The *maximin difference relation* $\succsim_{MD}$ on $\mathbb{R}^{2N}$ is a binary relation such that for any $(a, s), (b, s) \in \mathbb{R}^{2N}$,

$$(a, s) \succsim_{MD} (b, s) \text{ if and only if } \min_{i \in \mathcal{N}} (a - s)_i \geq \min_{i \in \mathcal{N}} (b - s)_i.$$

The maximin difference relation maximises the minimum difference in welfare relative to the status quo.⁴

⁴Note that the maximin difference relation, though s -Complete, is otherwise incomplete because it does

Theorem 1. (Liberal Non-Interference and the Maximin Difference Relation). *A contextualised social welfare ranking satisfies s -IDP, s -Weak Pareto Efficiency, s -Anonymity, s -Continuity, and s -Unit Comparable Cardinal Measurability if and only if it is the maximin difference relation.*

Proof. For the ‘if’ part, it is immediate to see that $\succsim_{MD}$ is a contextualised social welfare ranking that satisfies s -Weak Pareto Efficiency, s -Anonymity, and s -Unit Comparable Cardinal Measurability. To see that it satisfies s -IDP, consider any $(a, s), (b, s), (a', s), (b', s) \in \mathbb{R}^{2N}$ such that for some $j \in \mathcal{N}$,

$$a'_j < a_j \text{ and } b'_j < b_j$$

and $a'_i = a_i$ and $b'_i = b_i$ for all $i \neq j$. If $(a, s) \succ_{MD} (b, s)$, then $\min_{i \in \mathcal{N}}(a - s)_i > \min_{i \in \mathcal{N}}(b - s)_i$. Because $a_j > a'_j$, $b_j > b'_j$, and $a'_j > b'_j$, we have $\min_{i \in \mathcal{N}}(a' - s)_i > \min_{i \in \mathcal{N}}(b' - s)_i$. That is, $(b', s) \not\succ (a', s)$.

To see that $\succsim_{MD}$ satisfies s -Continuity, take any $(a, s) \in \mathbb{R}^{2N}$. Take any $(c, s) \in \{(x, s) \in \mathbb{R}^{2N} \mid (x, s) \prec_{MD} (a, s)\}$. We have, $\min_{i \in \mathcal{N}}(a - s)_i > \min_{i \in \mathcal{N}}(c - s)_i$. Let $\delta := \min_{i \in \mathcal{N}}(a - s)_i - \min_{i \in \mathcal{N}}(c - s)_i > 0$. Take any $(d, s) \in \mathbb{R}^{2N}$ such that $\|(c, s) - (d, s)\| < \delta$. We have, $\min_{i \in \mathcal{N}}(d - s)_i < \min_{i \in \mathcal{N}}(a - s)_i$ so that $B((d, s), \delta) \subseteq \{(x, s) \in \mathbb{R}^{2N} \mid (x, s) \prec_{MD} (a, s)\}$. A similar argument shows that $\{(x, s) \in \mathbb{R}^{2N} \mid (x, s) \succ_{MD} (a, s)\}$ is open.

For the ‘only if’ part, consider any $\succsim$ that is a contextualised social welfare ranking which satisfies s -IDP, s -Weak Pareto Efficiency, s -Anonymity, s -Unit Comparable Cardinal Measurability, and s -Continuity. By s -Completeness, it is enough to show that for any $(a, s), (b, s) \in \mathbb{R}^{2N}$, $(a, s) \succ (b, s)$ implies $(a, s) \succ_{MD} (b, s)$, and $(a, s) \sim (b, s)$ implies $(a, s) \sim_{MD} (b, s)$. Take $(a, s), (b, s) \in \mathbb{R}^{2N}$ arbitrary. It is without loss of generality to consider s such that $s = s^\pi$ for all permutations π . To see this, by s -Unit Comparable Cardinal Measurability, $(a, s) \succsim (b, s)$ if and only if $(\tau(a), \tau(s)) \succsim (\tau(b), \tau(s))$ for all unit-comparable positive affine transformations. This means we may focus on the specific τ given by $\tau_i(x) = x_i + \beta_i$ where $\beta_i = \max_{k \in \mathcal{N}} s_k - s_i$ for all $i \in \mathcal{N}$ and $x \in \mathbb{R}^N$. By construction, $\tau(s) = \tau^\pi(s)$ for all permutations π . Moreover, by s -Anonymity, s -Transitivity, and noting that s is invariant to permutations, it is without loss of generality to reorder the components in a and b in ascending order so that $a_k \leq a_{k+1}$ and $b_k \leq b_{k+1}$ for all $k = 1, \dots, N - 1$.

First, we show that if $(a, s) \succ_{MD} (b, s)$, then $(a, s) \succ (b, s)$. Suppose by contradiction that $(a, s) \succ_{MD} (b, s)$, but $(b, s) \succ (a, s)$. Because $(a, s) \succ_{MD} (b, s)$, we have,

$$(a - s)_k \geq \min_{m \in \mathcal{N}}(a - s)_m > \min_{m \in \mathcal{N}}(b - s)_m =: (b - s)_1$$

not rank profiles that have different status quos. This means that the maximin difference relation is formally equivalent to a family of SWRs $\{\succsim_s\}_{s \in \mathbb{R}^N}$ indexed by the status quo in an unordered way such that $x \succsim_s y$ if and only if $\min_{i \in \mathcal{N}} x_i - s_i \geq \min_{i \in \mathcal{N}} y_i - s_i$.

for all $k \in \mathcal{N}$. In particular, $(a - s)_1 > (b - s)_1$ so that $a_1 > b_1$.

Step 1: Because $(b, s) \succ (a, s)$, by *s-Weak Pareto Efficiency*, $b_j \geq a_j$ for some $j \in \mathcal{N}$. Therefore, $b_1 < a_1 \leq a_j \leq b_j$. Define $(a^*, s), (b^1, s) \in \mathbb{R}^{2N}$ to be such that, for some $\gamma, \delta > 0$,

$$a_1^* := a_1 - \gamma \in (b_1, \infty) \quad \text{and} \quad b_j^1 := b_j - \delta \in (a_1^*, a_j)$$

with $a_k^* = a_k$ for all $k \neq 1$ and $b_k^1 = b_k$ for all $k \neq j$. Consider a permutation π_{1j} that swaps the components 1 and j . By *s-Anonymity*, we have $(b^{\pi_{1j}}, s^{\pi_{1j}}) \sim (b, s) \succ (a, s)$, where

$$a_1^* < a_1 \quad \text{and} \quad (b^1)_1^{\pi_{1j}} < b_1^{\pi_{1j}} \quad \text{and} \quad (b^1)_1^{\pi_{1j}} > a_1^*.$$

By *s-IDP*, *s-Completeness*, *s-Anonymity*, and *s-Transitivity*, $(b^1, s) \sim ((b^1)^{\pi_{1j}}, s^{\pi_{1j}}) \succsim (a^*, s)$ where $a_1^* > b_1^1$ and $a_j^* > b_j^1$.

Step 2: Let $\eta > 0$ be such that $\eta < \min_{m \in \mathcal{N}} \{a_m^* - b_m^1 \mid a_m^* > b_m^1\}$. Denote $a^1 \in \mathbb{R}^N$ to be such that $a_k^1 = a_k^* - \eta$ for all $k \in \mathcal{N}$. By *s-Weak Pareto Efficiency*, $(a^*, s) \succ (a^1, s)$, and by *s-Transitivity*, $(b^1, s) \succ (a^1, s)$. If $a_k^1 > b_k^1$ for all $k \in \mathcal{N}$, then this contradicts *s-Weak Pareto Efficiency*. So $b_\ell^1 \geq a_\ell^1$ for some $\ell \notin \{1, j\}$ and $(b^1, s) \succ (a^1, s)$.

We may repeat Steps 1 and 2 finitely many times until we reach some $n \in \mathbb{N}$ such that $a_k^n > b_k^n$ for all $k \in \mathcal{N}$ which contradicts *s-Weak Pareto Efficiency*. Therefore, $(a, s) \succsim (b, s)$ whenever $(a, s) \succ_{MD} (b, s)$. It remains to show that $(a, s) \sim (b, s)$ is impossible.

Suppose by contradiction that $(a, s) \sim (b, s)$. Let $\varepsilon > 0$ be such that $(a - s)_1 - \varepsilon > (b - s)_1$. Denote $a^\varepsilon \in \mathbb{R}^N$ to be such that $a_k^\varepsilon = a_k - \varepsilon$ for all $k \in \mathcal{N}$. By construction, $(a^\varepsilon, s) \succ_{MD} (b, s)$. Using the same argument as above, we have, $(a^\varepsilon, s) \succsim (b, s)$. However, by *s-Weak Pareto Efficiency* and *s-Transitivity*, $(b, s) \sim (a, s) \succ (a^\varepsilon, s)$, a contradiction. Therefore, $(a, s) \succ (b, s)$ whenever $(a, s) \succ_{MD} (b, s)$.

Finally, we show that if $(a, s) \sim_{MD} (b, s)$, then $(a, s) \sim (b, s)$. By construction, $(a - s)_1 = (b - s)_1$. Suppose by contradiction that $(a, s) \succ (b, s)$. By *s-Continuity*, we have that $(a^\zeta, s) \succ (b, s)$ where $a_k^\zeta := a_k - \zeta$ for all $k \in \mathcal{N}$ for some $\zeta > 0$ small enough. However, by construction, $(a^\zeta, s) \prec_{MD} (b, s)$, a contradiction. An analogous argument rules out $(a, s) \prec (b, s)$. $\square$

Theorem 1 shows that our liberal principle of non-interference, together with standard efficiency, fairness, continuity, and informational properties, characterises the maximin difference relation.

Mariotti and Veneziani (2009) show for standard SWRs that the Individual Damage Principle, together with Strong Pareto Efficiency, and Anonymity, fully determines the social ranking, yielding the discontinuous leximin criterion. The present result differs by relaxing Strong Pareto Efficiency to Weak Pareto Efficiency. In doing so, Continuity is no longer incompatible with the characterisation; instead, it selects the continuous maximin

difference relation by ruling out discontinuous refinements. Lombardi et al. (2016) also relax Strong Pareto Efficiency to Weak Pareto Efficiency, finding for standard SWRs that the Individual Damage Principle, together with Weak Pareto Efficiency, Anonymity, and Continuity, characterises the standard maximin SWR.

Proposition 1 and Theorem 1 show that when a status quo is taken into account, the Individual Damage Principle implies an egalitarianism with respect to welfare differences instead of welfare levels as for standard SWRs. For contextualised SWRs, unit-comparable cardinal measurability of welfare is a strict lower bound on information about welfare to escape impossibilities under liberal non-interference. Under this bound, the Individual Damage Principle gives absolute priority to individuals who are worst-off based on their welfare differences relative to the status quo.

Equalising welfare differences is equivalent to equalising welfare levels only in a special case where the status quo $s_i = s_j$ for all $i, j \in \mathcal{N}$. Only in this case, the maximin difference relation is equivalent to the s -maximin SWR given by $\succsim_{sM}$ such that for all $(a, s), (b, s) \in \mathbb{R}^{2N}$, $(a, s) \succsim_{sM} (b, s)$ if and only if $\min_{i \in \mathcal{N}} a_i \geq \min_{i \in \mathcal{N}} b_i$. The s -maximin SWR, though satisfying the s -Individual Damage Principle, s -Weak Pareto Efficiency, s -Anonymity, and s -Continuity, violates s -Unit-Comparable Cardinal Measurability. Examples 3.1 to 3.4 demonstrate the independence of the remaining properties used in Theorem 1. Example 3.1 violates only the s -Individual Damage Principle.

Example 3.1. For any $(a, s), (b, t) \in \mathbb{R}^{2N}$,

$$(a, s) \succsim (b, t) \text{ if and only if } \max_{i \in \mathcal{N}} (a - s)_i \geq \max_{i \in \mathcal{N}} (b - t)_i.$$

Example 3.2 violates only s -Weak Pareto Efficiency

Example 3.2. For any $(a, s), (b, t) \in \mathbb{R}^{2N}$,

$$(a, s) \sim (b, t).$$

Example 3.3 violates only s -Anonymity.

Example 3.3. For any $(a, s), (b, t) \in \mathbb{R}^{2N}$ and for some $i \in \mathcal{N}$,

$$(a, s) \succsim (b, t) \text{ if and only if } a_i \geq b_i.$$

Example 3.4 violates only s -Continuity.

Example 3.4. For any $(a, s), (b, t) \in \mathbb{R}^{2N}$, we have $(a, s) \succsim (b, t)$ if and only if either $\overline{a - s} = \overline{b - t}$, or there is some $k \in \{1, \dots, N\}$ such that $(\overline{a - s})_k > (\overline{b - t})_k$ and $(\overline{a - s})_j = (\overline{b - t})_j$ for all $j < k$.

4 Conclusion

So what are the implications of the Individual Damage Principle when a status quo is taken into account? We show that this weak liberal principle of non-interference that only protects the rights of an individual against being punished when they have already suffered a strict welfare loss while no one else was harmed, together with standard efficiency, fairness, continuity, and informational properties, characterises the maximin difference relation that gives absolute priority to individuals who are worst-off relative to the status quo. The maximin difference relation implies an equalisation of welfare differences relative to the status quo, instead of the equalisation of welfare levels as for standard SWRs. Equalising welfare levels is recovered only as a special case when all individuals have the same welfare at the status quo.

Egalitarianism with respect to welfare differences implies a strong protection of individual baselines by guaranteeing each individual a veto against any move away from the status quo whenever such a move would lead them to a welfare loss, regardless of the potential welfare gains to others. Moreover, in applications where welfare levels are interpersonally comparable, equalising welfare *differences* can even preserve pre-existing inequalities in welfare *levels*.

There are a number of directions for further work in incorporating the historical context into normative evaluation. Given that the maximin difference relation does not compare profiles under different status quos, it is immediate that it violates standard stationarity properties translated into a static setting in that $(x_{t+1}, x_t) \succsim (x'_{t+1}, x_t)$ neither implies nor is implied by $(x_{t+2}, x_{t+1}) \succsim (x_{t+2}, x'_{t+1})$ for all $x_k, x'_k \in \mathbb{R}^N$ and $k \in \mathbb{N}$. However, this does not mean that liberal non-interference principles are incompatible with stationarity, nor related properties such as dynamic consistency, in an intertemporal setting where the primitive is instead a family of SWRs $\{\succsim_t\}_{t \in \mathbb{N}}$. Studying liberal non-interference principles in an intertemporal setting requires an extension of the framework used.

Moreover, Lecouteux and Mitrouchev (2024) propose that context-dependent normative evaluation should satisfy a *self-determination criterion*. This criterion states that given any individual i , and any two contexts C and C' , we have that C is normatively preferred to C' when the set of known motivations influencing i under C is larger than that under C' . Contextualised SWRs are defined in a framework of welfarist social evaluation which does not account for non-welfarist information over which the self-determination criterion holds. Incorporating information about the context in terms of liberties, functionings, or capabilities, among many other non-welfaristic dimensions, would require a broader framework for normative evaluation.

Declarations

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